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THE DESIGN OF SUPERMANEUVERABLE FIGHTER AIRCRAFT, HIGH-PRECISION SPACE-BORN OPTICAL TRACKING SYSTEMS AND TRANSATMOSPHERIC HYPERVELOCITY VEHICLES WILL POSE SIGNIFICANT CHALLENGES TO MODERN CONTROL SYSTEM DESIGN THEORY. THE THEME OF THE RESEARCH HAS BEEN "MAKING MODERN CONTROL THEORY WORK." THE PRODUCT OF THE RESEARCH HAS BEEN THEORY, ALGORITHMS AND SOFTWARE APPLICABLE TO MULTIVARIABLE FEEDBACK CONTROL PROBLEMS IN WHICH THERE ARE DESIGN CONSTRAINTS REQUIRING ROBUST ATTAINMENT OF STABILITY AND CONTROL PERFORMANCE OBJECTIVES IN THE FACE OF BOTH STRUCTURED AND UNSTRUCTURED UNCERTAINTY.

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FINAL REPORT

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PRACTICAL METHODS FOR ROBUST MULTIVARIABLE CONTROL

July 15, 1985 - July 14, 1988

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ABSTRACT

The design of supermaneuverable fighter aircraft, high-precision space-born optical tracking systems and transatmospheric hypervelocity vehicles will pose significant challenges to modern control system design theory. The theme of the research has been "making modern control theory work." The product of the research has been theory, algorithms and software applicable to multivariable feedback control problems in which there are design constraints requiring robust attainment of stability and control performance objectives in the face of both structured and unstructured uncertainty.

CONTENTS

*E. A. Jonckheere and R. Li, "H^∞ Control Over a Planar Domain (Extended Abstract)," to appear, Proc. IEEE Conf. on Decision and Control, Austin, TX, December 7-9, 1988.

*M. G. Safonov and D. J. N. Limebeer, "Simplifying the H^∞ Theory via Loop-Shifting," to appear, Proc. IEEE Conf. on Decision and Control, Austin, TX, December 7-9, 1988.

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INTRODUCTION: THE PROBLEM

The underlying problem in robust feedback control system synthesis is to find a feedback controller C(s) such that a given vector, say col (e,u,y), whose components comprise the control system's error, control and plant output signals, respectively, remains in a specified set despite uncertain disturbances, parameters, gains, phases and nonlinearities within a given set, say $\underline{D}$. The performance specifications on the signals e, u, and y may be expressed in terms of frequency response inequalities (for broadband r.m.s. disturbance rejection), closed-loop pole locations (for acceptable transient response to impulsive and step disturbances), closed-loop zero locations (for asymptotic tracking and asymptotic rejection of disturbances with known poles).

It turns out that this general problem can be reformulated as a consequence a certain lemma of Youla as the problem of finding the set, say $\underline{X}$, of all transfer function matrices X(s) having "stable" poles (i.e., poles in a stipulated region) for which the <u>excess stability margin</u> k_m satisfies

$$k_m(A + BXC; \underline{D}) > 1 \qquad (1)$$

(see [22,23] and the references therein). Here the A(s), B(s), and C(s) are transfer function matrices which depend on the specific plant and on where the uncertain noises, parameters, etc., enter. The function $k_m(T;\underline{D})$ is defined for any transfer function matrix T(s) and any set $\underline{D}$ of uncertain operators as [40]

$$k_m(T;\underline{D}) = \inf \{k\text{: } k \text{ real, } (I + kDT)^{-1} \text{ is "unstable" for some D in the set } \underline{D}\}\text{ ;} \qquad (2)$$

the quantity $1/k_m$ has been called the <u>structured singular value</u> $\mu(T)$ by Doyle [26]. Thus, $k_m(T;\underline{D})$ is the gain margin (for the worst-case D in the set $\underline{D}$) of a hypothetical feedback system having loop transfer function T. The quantity $k_m(T,\underline{D})$ is defined to be zero when T is open loop unstable. The notion of "unstable" is left intentionally vague here, since the appropriate definition of stability may vary depending on the application. For example, it may refer to stability with a specified degree, e.g., with all poles in some specified set [58]. A "stable" function X(s) (that is, a stabilizing compensator C(s)) verifying (1) achieves the ultimate design objective, but one may also look at cptimizing the performance as

$$k_m^{opt} := \max_{X \text{ "stable"}} k_m(A + BXC;\underline{D}) . \tag{3}$$

Currently, the function $k_m(.\,;.\,)$ can be computed only in special cases such as when the set $\underline{D}$ is finite or when $\underline{D}$ is the set of all transfer function matrices whose largest singular value is bounded for all frequency by a given number, i.e., when $||D||_\infty$ is bounded, in which case the problem (3) reduces to the multivariable L^∞ optimization problem [22,23]

$$k_m^{opt} := \min_{X \text{ "stable"}} || A + BXC ||_\infty . \tag{4}$$

The problem of developing a useful characterization of the set $\underline{X}$ of transfer function matrices X(s) satisfying (1) likewise can only be solved in special cases, e.g., $\underline{D}$ singular-value bounded or $\underline{D}$ *real*, scalar gains. Also unsolved, and not less difficult, is the problem of optimizing the k_m-performance as described by (3). Our research over the past two and one-half years has addressed these unsolved problems, building upon and extending the theoretical base of L^∞ optimal control theory. We have made significant strides toward our goal of creating a cohesive body of theory that may be used by engineers to solve the broadest possible class of practical robust multivariable feedback control design problems.

SUMMARY OF PROGRESS PREVIOUSLY REPORTED

Since research under AFOSR Grant 85-0256 began two and one-half years ago in July 1985, progress has been made on several aspects of this problem, leading to a substantial number of AFCSR-supported reports and publications [1-20, 24, 28-31, 34, 56-61]. Among the new results is a vastly improved "Toeplitz + Hankel" algorithm for computing the minimal cost for L^∞ optimal control problems [3,5,14,16,17]; the results promise to reduce computer-time for L^∞ control calculatios by a factor of 10. Another result [18] involves a vector-valued alternative to the standard L^∞ control problem which has been bound to enable a more precise trade-off between sensitivity S(s) and complementary sensitivity I-S(s). In [4,5] we describe how the frequency-weighted LQG (Linear Quadratic Gaussian) synthesis theory (Safonov et al. [25]) was used to design a robust multivariable controller for a 40-state model of a flexible mechanical truss structure; the control design worked well when digitally implemented and connected to the infinite-order real system. In [2] a homotopy method for eliminating conservativeness in $\mu(T;D)$ stability margin calculation was developed and evaluated, but found to be too computationally demanding to be practical. Further study resulted in a significant breakthrough in nonconservative $\mu(T;D)$ calculation techniques in [1,7,19]; *these new results make computation of $\mu(T;D)$ practical for the first time for the important* case when the set $\underline{D}$ is a cube in $\mathbb{R}^n$ (i.e., the case of several uncorrelated unknown-but-bounded uncertain real parameters); this problem has become popularly known as the "real k_m" or "real μ" problem. A major practical advance in 1986 was the development at USC of a software package [8] within the CTRLC^{TM}/PC-MATLAB^{TM} framework for solving a broad class of L^∞ optimal control problems. Over the past year, in further work not supported by AFOSR, we have collaborated with the publishers of PC-MATLAB to create a new PC-MATLAB Robust-Control

Toolbox, software package and user's guide [65]. Our toolbox makes the L^∞ optimal control theory and associated Hankel and balanced model reduction theory widely accessible to practicing engineers.

The process of developing and testing this software enabled us to identify and resolve a number of minor, but critical, shortcomings of the existent L^∞ conceptual algorithms; the initial versions of the refined L^∞ theory and algorithms were summarized in [15]. An early version of our Robust-Control Toolbox called LINF was used for a "benchmark" multivariable aircraft controller design problem in [9] and for a flexible space structure controller design in [65]. In a separate development, we developed a significantly improved computer-oriented criterion for nonlinear stability which may render the celebrated Popov criterion obsolete; our new nonlinear stability criterion is superior (i.e., less conservative than) the standard graphical criteria including the circle criterion, the off-axis circle criterion, and the Popov criterion. Another major breakthrough has been the solution of the diagonally-scaled L^∞ optimal control problem for a limited but nontrivial class of problems [10,12,30]; this new theory enables achievement of our ultimate design objective, namely the solution of (3) for a limited class of problems involving complex structured uncertainty.

PROGRESS THIS YEAR

Since July 15, 1987 we have made several major advances in the area of H^∞ optimal control theory, in algorithms for model order reduction and in the mathematical system theory. We regard the first two of the following to be major practical advances, and the third has been a major theoretical advance:

1. Two-Riccati H^∞ Formulae [36, 56, 60]
2. Basis-Free Model-Reduction Formulae [24, 28, 57]

3. Spectral Theory of LQ and H^∞ Problems [59-61]

4. H^∞ Control Over Arbitrary Regions of the Complex Plane [58]

Two-Riccati H^∞ controller formula, developed largely independently by Doyle et al. [38, 39, 63, 64] and by Limebeer, Kasenally and Safonov [56, 62] and closely related to the formula of Juang and Jonckheere [35, 36], constitute what may be the single greatest breakthrough in control theory in the past decade. These formula enable one to completely bypass the Youla parameterization A + BXC and solve the multivariable L^∞ optimization problem 4 by solving two Riccati equations of the state-space (A, B, C, D) matrices of the plant. The result is the two-Riccati formula for "order n" H^∞ controllers which are no more complicated to compute or implement than H^2 controllers (i.e., LQG controllers). We have coded these formula using PC-MATLAB and found them to be superior for computer implementation of H^∞ optimal control theory, producing H^∞ controller solutions reliably for plants with dozens of states in only a few minutes of computer time on a VAX 11/780 and on a SUN 3/50 workstation.

We pursue the "two-Riccati" breakthrough in the H^∞ theory further in [60,61]. In [60], we develop an embedding technique involving "loop shifting" variable changes which enable the general H^∞ optimal control problem to be reduced to the much simpler special case intially treated by Doyle et al. [38, 39, 63]. The simplications made possible by our loop shifting techniques made it practical, for the first time, to present complete derivations of the H^∞ theory for the general case. In computer studies we have also observed that the loop-shifting formula are easier to code and slightly faster to compute with than the two-Riccati general formulae of Glover et al. [64] and Limebeer et al. [56].

The second major advance, our basis free model reduction formulae [24, 28, 57], has made model order reduction with an infinity-norm error-criterion practical for those systems which stand to benefit the most from model reduction, viz., systems with some modes which are nearly u[illegible]ntrollable or nearly unobservable. Though perhaps not particularly exciting from a purely theoretical point of view, they are a major advance because they make Hankel Optimal (HO) model reduction, Balanced Truncation (BT) model reduction and Balanced Stochastic Truncation (BST) model reduction p-actical. A critical shortcoming of these three methods that had gone unnoticed by theoreticians heretofore was that they simply did not work on systems with uncontrollable or unobservable modes. The first step in all the literature in these infinity-norm criterion model reduction methods involved finding a "balancing transformation," a transformation which generically fails to exist for non-minimal realizations. Theoreticians failed to recognize the problem since, in theory, one can always eliminate non-minimal modes. In practice, however, systems are generically observable and controllable, even if only barely so, and, in practice, one of the primary uses of model reduction is to identify and discard the barely observable/controllable modes. Moreover, a computer with finite numerical precision cannot distinguish a barely observable mode from an unobservable one and, in any case, some "barely observable" modes can turn out to have a very significant impact on the frequency-response of a system. Thus, it is folly to suppose, as theoreticians had, that one can usefully begin a model reduction procedure by discarding the unobservable and uncontrollable modes. Our basis-free methods for model reduction bypass the inherently ill-conditioned initial balancing step. The resulting model reduction formula are simpler, faster to compute, and most importantly they work. They work even for nonminimal and nearly nonminimal systems, reliably eliminating the unobservable and uncontrollable modes while ensuring that the important infinity-norm error bounds associated with Hankel, balanced truncation and balanced stochastic truncation model reduction methods are satisfied.

The relative-error infinity-norm error bounds of BST makes our basis free BST algorithm in [57] especially attractive for robust control system design. A "robustness theorem" [57] establishes that a model is useful for designing feedback control systems only if its relative error is less than one thoughout the control loop bandwidth as determined from singular-value Bode plots of the loop transfer function matrix. This robustness theorem proved vital in our TRW-supported large space-structure design study [65] in which a 4-state plant model surprisingly was proved to be adequate for a structure having 116 modes within the control loop bandwidth. This work is a spinoff of the so-called "phase matching" problem initiated by Jonckheere; see, e.g., [48], [49] and references therein.

The "Toeplitz + Hankel" operator theoretic interpretation of the H^∞ theory has led to a number of theoretical insights into the H^∞ optimal control problem which we hope will eventually lead us to better and faster computational algorithms and, perhaps, to generalization of the H^∞ control theory. Moving beyond our early work on fast Toeplitz + Hankel algorithms [3,14,16,17,34,35], our recent work in [59,61] achieves, we feel, a complete understanding of the links between the H^∞ problem and the spectral theory of the linear-quadratic problem. In a few words, this is the essence of the results in [59,61]:

Consider the standard 2-block frequency response inequality

$$\left\| \begin{matrix} H(j\omega) - Q(j\omega) \\ V(j\omega) \end{matrix} \right\| \leq \varepsilon, \ \forall \omega$$

verified for some

$$Q \in {H^\infty}_\perp$$

where

$$\begin{pmatrix} H(s) \\ V(s) \end{pmatrix} = \begin{pmatrix} D_H \\ D_V \end{pmatrix} + \begin{pmatrix} C_H \\ C_V \end{pmatrix} (sI - A)^{-1}B \in H^\infty$$

The key idea is to map the frequency response inequality to the time domain using Parseval's like arguments. This yields

$$\int_{-\infty}^{0} (x^T \ u^T) \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \leq \varepsilon^2 \int_{-\infty}^{0} u^T u \ dt, \ \forall \ u$$

where x is generated by the state space equation

$$\dot{x} = Ax + Bu$$

and

$$Q = -C^T{}_H C_H$$

$$R = D^T{}_V D_V$$

$$S = (Y_H + Y_V)B_H + C^T{}_V D_V$$

where

$$A^T(Y_H + Y_V) + (Y_H + Y_V)A = -(C^T{}_H C_H + C^T{}_V C_V)$$

The cornerstone of the spectral theory of the linear quadratic problem -- proved ten years ago by Jonckheere and Silverman -- is that

$$\int_{-\infty}^{0} (x^T \ u^T) \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} = (u_1, (T + H^*{}_1 H_2)u)$$

where T is Toeplitz and H is Hankel.

Using the LQ-H^∞ mapping, all the results of the spectral theory of the linear quadratic problem have an H^∞ interpretation, and vice-versa. Consequently, this symbiotic LQ/H^∞ theory has allowed to provide simple linear-qudratic insight to such problems as (i) degree of H^∞ compensator; (ii) pole/zero cancellation at H^∞ optimality; (iii) Riccati equation solution to H^∞ design; (iv) γ-iteration, etc. The challenge before us, now that we understand these relationships, will be to turn these operator-theoretic insights into practical algorithms. This is one of the aims of our current work.

The most significant practical impact of this symbiotic LQ/H^∞ theory, which we were first to introduce [3], is a better understanding of the termination condition on the σ-iteration. Indeed, in the 2-Riccati solution to the 4-block problem, the tolerance level γ is recursively decreased until "something" breaks down in the Riccati construction of the compensator achieving the tolerance γ. With this LQ/H^∞ theory at hand, we relate the breakdown of the 2-Riccati equation construction of the compensator to the spectral structure of several "Toeplitz + Hankel" operators. Depending on whether γ hits the continuous or discrete spectrum, the Riccati solution either has closed loop poles on the $j\omega$-axis or has the wrong sign. Finally, if optimality is achieved at the discrete spectrum, an easy procedure for reducing the size of the H^∞ compensator emerges.

Finally, we briefly discuss the fourth area in which we have made significant progress this past year: H^∞ control over a planar domain [58]. This work, which builds upon the PI's earlier work in [21,67] provides <u>state-space</u> formula for solving "one-block" H^∞ optimization problems over a subset Ω of the complex plane specifiable in the form

$$\Omega = \{z \in \mathbb{C} \mid \sum_{ij} \gamma_{ij} \bar{z}^i z^j \geq 0\}.$$

There are some technical conditions on the γ_{ij}'s which are not expanded upon here. Work is still in progress on this problem, but the key feature of the results that have emerged thus far is that the generalization from the usual left-half plane $\mathbb{C}_-$ can be handled via a simple modification of the controllability observability Lyapunov equations which determine the H^∞ optimum in conventional H^∞ problems in which Ω equals $\mathbb{C}_-$.

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